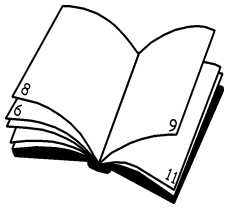
Warm-Up 1

1. _____ What is the least common multiple of 6, 8 and 10?

2. _____ A 16-page booklet is made from a stack of four sheets of paper that is folded in half and then joined along the common fold. The 16 pages are then numbered from front to back, starting with page 1. What are the other three page numbers on the same sheet of paper as page 5?



3. _____ What is the least natural number that has exactly three factors?

4. _____ What integer on the number line is closest to -132.48?

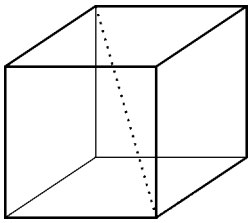
5. _____ Each side of hexagon ABCDEF has a length of at least 5 cm and AB = 7 cm. How many centimeters are in the least possible perimeter of hexagon ABCDEF?

6. _____ Walker Middle School sells graphing calculators to raise funds. The school pays $90 for each calculator and sells them for $100 apiece. They hope to earn enough money to purchase an additional classroom set of 30 calculators. How many calculators must they sell to reach their goal?

7. _____ Two different natural numbers are selected from the set {1,2,3,…,6}. What is the probability that the greatest common factor of these two numbers is one? Express your answer as a common fraction.

8. _____ School uniform parts are on sale. The $25 slacks can be purchased at a 20% discount and the $18 shirt can be purchased at a 25% discount. What is the total cost, in dollars, of three pairs of slacks and three shirts at the sale price, assuming there is no sales tax? Express your answer as a decimal to the nearest hundredth.

9. _____ A space diagonal of a polyhedron is a segment connecting two non-adjacent vertices that do not lie on the same face of the polyhedron. How many space diagonals does a cube have?



10. _____ What is the mean of $\frac{1}{2}$ and $\frac{7}{8}$? Express your answer as a common fraction.

Warm-Up 1

Answers

1.	120	(C, T, F)	5.	32	(M, C, F)	8.	100.50	(C, F)
2.	6, 11, 12	(S, M, P, T)	6.	270	(C, F)	9.	4	(M)
3.	4	(G, T, C, E)	7.	$\frac{11}{15}$	(T, M)	10.	$\frac{11}{16}$	(C, F)
4.	-132	(M, C)						

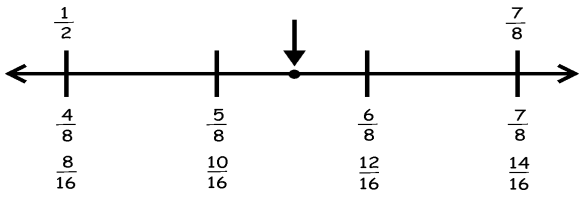
Solution – Problem #7

To find all of the possible combinations of two numbers that could be selected, let's make a chart. Make sure not to include situations twice (like choosing 1 & 2 as well as 2 & 1) or situations where the same number is used for both choices (like 2 & 2). To eliminate these options, they have been shaded gray in the chart. Notice there are 15 possible combinations (shown as white rectangles), and those where the greatest common factor is 1 are marked with an X; there are 11 of these. Therefore the probability is $\frac{11}{15}$.

	1	2	3	4	5	6
1		X	X	X	X	X
2			X		X	
3				X	X	
4					X	
5						X
6						

Representation – Problem #10

This problem can be modeled geometrically by finding the point on a number line equidistant from $\frac{1}{2}$ and $\frac{7}{8}$. If $\frac{1}{2}$ is renamed as $\frac{4}{8}$, it is easy to see that each section of the number line is $\frac{1}{8}$ units long, but the middle is still not exactly known. Changing the denominators to 16, though, will show that the middle is halfway between $\frac{10}{16}$ and $\frac{12}{16}$, which is $\frac{11}{16}$.



Connection to ... Rectangular prisms (Problem #9)

The cube in #9 is just a special rectangular prism. Due to the regular use of rectangular prisms in geometry problems, it is worth memorizing some of the formulas that go with them. For a rectangular prism with a length of x units, a width of y units and a height of z units, the *volume* is equal to the product $x \cdot y \cdot z$, the *surface area* is equal to $2xy + 2yz + 2xz$, and the *length of a space diagonal* is equal to $\sqrt{x^2+y^2+z^2}$. Notice, for any cube such as the figure in problem #9, the length of the space diagonal will be $\sqrt{x^2+x^2+x^2} = \sqrt{3x^2} = x\sqrt{3}$.

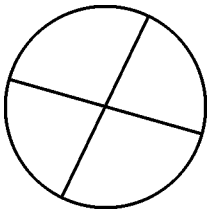
Warm-Up 2

1. _____ The square root of what number is double the value of 8?

2. _____ A hummingbird flaps its wings 1500 times per minute while airborne. While migrating south in the winter, how many times during a 1.5 hour flight does the hummingbird flap its wings? Express your answer in scientific notation.

3. _____ Suppose $\psi(a,b,c) = ab^c$. Compute $\psi(1,2,3) + \psi(3,2,1) + \psi(3,1,2)$.

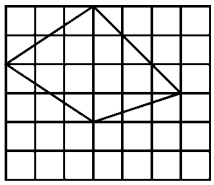
4. _____ A pizza with a diameter of 12 inches is divided into four slices as shown. The central angles for the two larger congruent slices each measure 20 degrees more than the central angles for each of the two smaller congruent slices. What is the measure, in degrees, of a central angle for one of the smaller slices?



5. _____ To determine whether a number N is prime, we must test for divisibility by every prime less than or equal to the square root of N. How many primes must we test to determine whether 2003 is prime?

6. _____ A farmer plants seeds for a 75-acre field of yellow sweet clover. A 25-pound bag of seed costs $24. How much would it cost, in dollars, to seed the field if twelve pounds of seed were used per acre?

7. _____ What is the area, in square centimeters, of the figure shown?



= 1 sq. cm.

8. _____ On a 25-question multiple choice test, Dalene starts with 50 points. For each correct answer, she gains 4 points; for each incorrect answer, she loses 2 points; for each problem left blank, she earns 0 points. Dalene answers 16 questions correctly and scores exactly 100 points. How many questions did she answer incorrectly?

9. _____ Which pair of the following expressions are never equal for any natural number x: x, x^2, 2^x, x^x?

10. _____ A five-digit number is called a *mountain number* if the first three digits increase and the last three digits decrease. For example, 35,763 is a mountain number but 35,663 is not. How many five-digit numbers greater than 70,000 are mountain numbers?

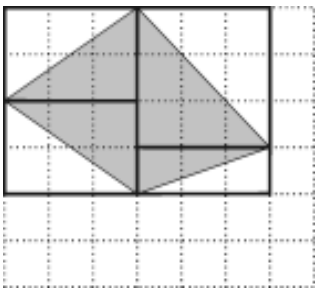
Warm-Up 2

Answers

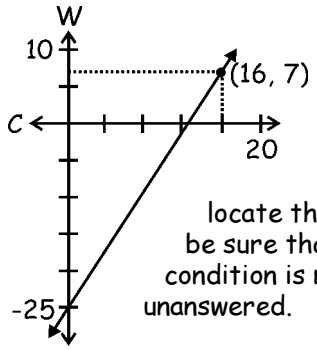
1.	256	(C)	5.	14	(T, C, E, G)	8.	7	(T, C, F, G)
2.	1.35×10^5	(C)	6.	864	(C)	9.	x, 2^x	(E, G, F, T)
3.	17	(F, C)	7.	12	(M, F, C, P)	10.	36	(T, P, E, S)
4.	80	(C, F, M)						

Solution – Problem #7

Separating the shape into 4 triangles, we see that each of the triangles is half of a rectangle. Therefore the area of the original region will be half of the largest rectangular region circumscribed about the shaded area. Just by counting, we can see that there are 24 square centimeters within the four small rectangular regions. Taking half of this amount yields the answer of 12 square centimeters for the area of the shaded region.



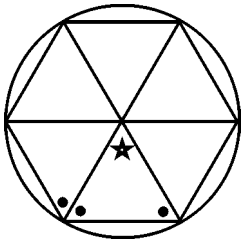
Representation – Problem #8



The situation in this problem can be represented with the equation Total Points = 50 + 4C – 2W, where C is the number of correct answers and W is the number of wrong ones. Since we are looking at the situation where Dalene earns 100 points, the equation we need to graph is 100 = 50 + 4C – 2W or W = 2C – 25. Since Dalene had 16 correct answers, look at the W-value on the graph when C = 16. On a graphing calculator, using the Table function or Trace function can help you locate the exact value for W when C = 16. We see that W = 7. Finally, we need to be sure that W + C < 25, since there are only 25 questions on the exam. This condition is met, and we can also determine now how many problems were left unanswered.

Connection to ... Angle measures in polygons (Problem #4)

Measuring the central angle in a circle can be used to find the angle measures of a regular polygon. A regular n-sided polygon can be inscribed in a circle. A regular hexagon is shown here. Notice that the central angle (star) is $\frac{360}{n}°$ for any regular n-gon. Since the triangles in the polygon are isosceles, the sum of the measures of the base angles (dots) is $(180 - \frac{360}{n})°$. An interior angle of the polygon is composed of two of these base angles, so its measure will also equal $(180 - \frac{360}{n})°$. Therefore, the measure of an interior angle of this regular hexagon is equal to $(180 - \frac{360}{6}) = 120°$.



Workout 1

1. _____ What integer on the number line is closest to $\frac{-169}{9}$?

2. _____ On Tuesday, the Beef Market sold 400 pounds of prime rib steak at $9.98 per pound and 120 pounds of rib-eye steak at $6.49 per pound. What was the average cost in dollars per pound of the steaks sold on Tuesday? Express your answer to the nearest hundredth.

3. _____ 

The earned run average (ERA) of a major league baseball pitcher is determined by dividing the number of earned runs the pitcher has allowed by the number of innings pitched, then multiplying the result by 9. What is Ray Mercedes' ERA, to the nearest hundredth, if he has pitched 164 innings and allowed 48 earned runs?

4. _____ An algebraic expression of the form $a + bx$ has the value of 15 when $x = 2$ and the value of 3 when $x = 5$. Calculate $a + b$.

5. _____ In 1994, the average American drank 60 gallons of soft drinks. How many ounces per day of soft drinks did the average American drink in 1994? There are 128 ounces in one gallon. Express your answer to the nearest whole number.

6. _____ Three consecutive prime numbers, each less than 100, have a sum that is a multiple of 5. What is the greatest possible sum?

7. _____ An oak rocking chair once owned by former President John F. Kennedy was sold in an auction for $442,500. This represents 8850% of its estimated value before the auction. How many dollars was the estimated pre-auction value?

8. _____ On her daily homework assignments, Qinna has earned the maximum score of 10 on 15 out of 40 days. The mode of her 40 scores is 7 and her median score is 9. What is the least that her arithmetic mean could be? Express your answer as a decimal to the nearest tenth.

9. _____ Paul earns an hourly wage of $28.80 and earns hourly benefits worth $8.11. What percent of Paul's earnings (wages & benefits) are his benefits? Express your answer to the nearest whole number.

10. _____ What is the greatest integer solution to $\pi x - 17 < 20$?

Workout 1

Answers

1.	-19	(C, M)	5.	21	(C)	8.	7.9	(T, E, C, L, S)
2.	9.17	(C, F)	6.	235	(T, G, L)	9.	22	(C)
3.	2.63	(C, F)	7.	5000	(C)	10.	11	(C, G)
4.	19	(C, T)						

Solution and Representation – Problem #8

Representing the information in the format below will help us find the missing scores. The top row of numbers indicates the order of the 40 scores, in ascending order. The bottom row of numbers is the actual scores we know. The stars represent the 15 scores of 10. For the median to be 9, the middle score, when all 40 scores are put in ascending order, must be 9. Since there is an even number of scores, the 20th and 21st scores are each 9, or they are 8 and 10. If the 21st score is a 10, then all of the scores 21-40 are 10's. We're hoping for smaller numbers to keep the arithmetic mean as small as possible. Therefore, we should go with 9 and 9.

```
1 2 3 4 5 6 7 8 9 0 1 2  3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0
                         9 9       * * * * * * * * * * * * * * *
```

Now we can also fill in the 22nd-25th scores as 9's, since we don't want any more 10's.

```
1 2 3 4 5 6 7 8 9 0 1 2  3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0
                         9 9 9 9 9 9 * * * * * * * * * * * * * * *
```

For the mode to be 7, there must be at least 16 scores of 7, since there were 15 scores of 10. Again, trying to keep the arithmetic mean as small as possible, we can put the 7's in the 4th-19th places and eliminate the possibility of any more 9's or 8's. Now we have the following information:

```
1 2 3 4 5 6 7 8 9 0 1 2  3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0
    7 7 7 7 7 7 7 7 7  7 7 7 7 7 7 7 9 9 9 9 9 9 * * * * * * * * * * * * * * *
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With this arrangement of scores we have met all of the criteria. The only thing left to do is fill in the remaining scores. Filling these in with 0's will lead us to the least arithmetic mean. The sum of Qinna's scores is 3(0) + 16(7) + 6(9) + 15(10) = 316. Dividing by 40 yields the arithmetic mean 7.9

Connection to ... Sports (Problem #3)

Many sports use mathematics to compare players' performances, to assess team performance or to determine rules to ensure fairness in deciding championships. A baseball player's batting average is a common statistic many people are familiar with. What is a slugging percentage in baseball? What is the difference between these two statistics? Check out some of the many Web sites devoted to exploring math concepts found in different sports.